Social network impact on persistence in a finite population dynamic seed exchange model

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MIRES: Méthodes Interdisciplinaires sur les Réseaux d'Échanges de Semences

Groupe financé par le département MIA de l'INRA regroupant statisticiens, modélisateurs déterministes, ethnobiologistes, écologues et généticiens.

2 axes:

- Modélisation de processus dynamiques tenant compte de l'organisation sociale des individus (réseau sociaux).
- Développement de procédures d'analyse de données hétérogènes mêlant données relationnelles et données génétiques.

https://sites.google.com/site/miresssna/home/presentation

Context: Emergence of an alternative agriculture model in France from 10 years: Réseau Semences Paysannes

Characteristics:

- people involved in seed autonomy
- **seed exchanges among farmers** and seed multiplication activities
- interest in old varieties of crop species
- small but growing community

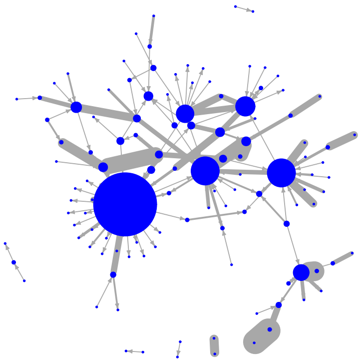
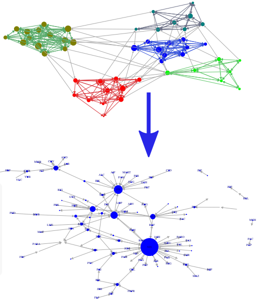


Figure: Seed exchange network among farmers involved in alternative agriculture

What are the properties of such system to maintain crop varieties?

Advice Network



Seed Exchanges network

Assumption

Seed exchange networks are nested within advice networks

Refine question

To what extent do the topological properties of the advice network influence the persistence of crop varieties?

Outline

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Dynamic Model specifications: assumptions

- number of farms=nodes=patches (n) is fixed in time
- each patch has two possible states: presence or absence of the variety (no demography, drift, mutation, selection, migration and recombination).
- **Initial state**: every patch is occupied.

Temporal dynamic : 2 steps

- **extinction**: each occupied patches may be affected with probability e,
- **colonisation**: for empty patches with rate c from an occupied neighbour based on a **fixed network G**.

Remark

This model is similar to SIS (Susceptible Infected Susceptible) in epidemiology. Studied in Gilarranz& Bascompte (2012), Chakrabarti (2008)).

Dynamic model: Illustration

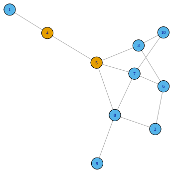
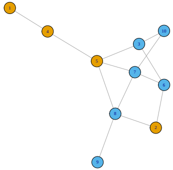


Figure: Generation t



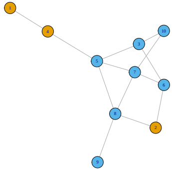
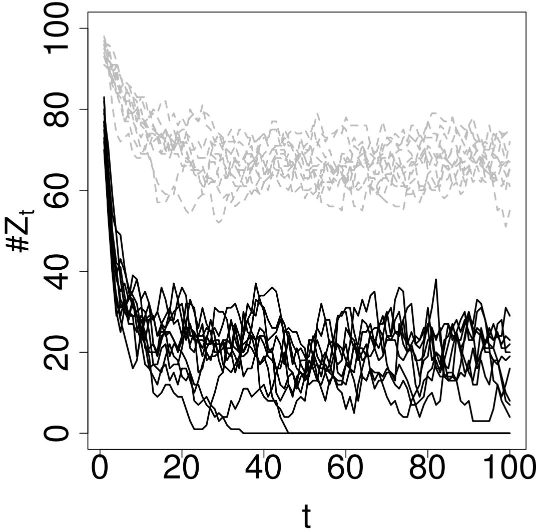


Figure: Generation $t+1$: extinction and colonisation

Assessing persistence under uncertainties



Equilibrium ?

- Model: $\{Z_t\}_{t\leq 0} \in \{0,1\}^N$: Markov chain with 2^N possible states.
- when N not too large (≤ 10), computing the transition matrix $M = E \cdot C$ (Day & Possingham (1995)).
- If $e > 0$, convergence of the chain toward its stationary distribution: a coffin state "total extinction":
- Extinction time:

$$T_0 = \inf\{t > 0, Z_t = 0\} \,,$$

$\mathbb{P}_z(T_0 < \infty) = 1$ for any initial state z .

Speed of convergence

$$\mathbb{P}_z(T_0 > t) = O(\lambda_{M,2}^t) \,,$$

where $\lambda_{M,2}$ is the second eigenvalue of M.

Quasi-equilibrium

- If $\mathbb{E}(T_0) >> nbgenerations \Rightarrow$ quasi-equilibrium.
- Z_t conditioned to $\{T_0 > t\}$ (non extinction) can converge toward a so-called quasi-stationary distribution
- If $\{Z_t\}_{t\geq 0}$ is irreducible and aperiodic ($\Leftrightarrow G$ has a unique connected component), existence and uniqueness of the quasi-stationary distribution (Darroch & Seneta, 1965).
- its transition matrix R is $2^n - 1 \times 2^n - 1$ obtained by deleting the first row and column of M.
- Convergence toward the quasi-stationary distribution is governed by $|\lambda_{R,2}|/\lambda_{R,1}$:

$$\sup_{z,z' \text{ transient states}} |\mathbb{P}_z(Z_t = z' | T_0 > t) - \alpha_{z'}| = O\left(\left(\frac{|\lambda_{R,2}|}{\lambda_{R,1}}\right)^t\right). \quad (1)$$

- quasi-stationary distribution is met if $|\lambda_{R,2}|/\lambda_{R,1} << \lambda_{R,1}$.

quantities of interest/to be monitored

Our choice, study 100 generations to make the comparisons:

- Probability of persistence in 100 generations: $\mathbb{P}(T_0 > 100)$.
- Mean number of occupied patches at the 100^{th} generation: $\mathbb{E}(\#Z_{100})$ or mean number of occupied patches at the 100^{th} conditioned to non extinction $\mathbb{E}(\#Z_{100} | T_0 > 100)$.

Sensitivy Analysis

$$e,\ c,\ G \rightarrow \boxed{\text{Dynamic Model}} \rightarrow \mathbb{P}(T_0 > 100),\ \mathbb{E}(\#Z_{100}),$$

based on:

- exact computations when the number of nodes ≤ 10,
- simulations otherwise, enhanced when necessary by particular or IS techniques.

Differences with deterministic models

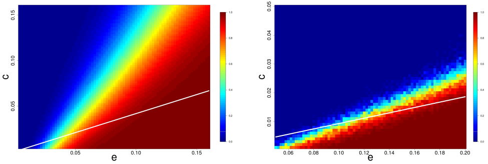


Figure: For fixed networks with 10 (lhs) and 100 nodes (rhs), Probabilities of extinction in 100 generation with varying e and c.

White line corresponds to the threshold

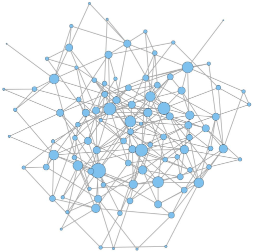
$$e/c = \lambda_{G,1} .$$

(Hanski & Ovaskainen (2000); Sole & Bascompte (2006))

When dealing with a finite horizon in time and a finite population, ratio e/c is not sufficient.

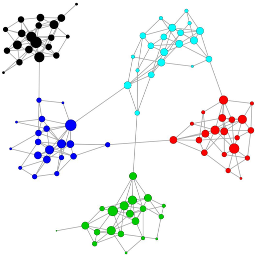
Outline

Random Graph: Erdős-Rényi model



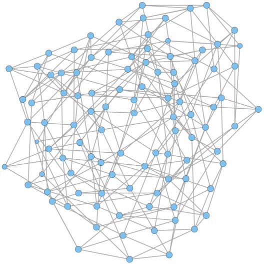
- Each pair of nodes has the same probability to be linked by an edge.
- Independence of edges.

Community model



- Groups with the same intra and inter connection probabilities and same size.
- Stronger intra connection than inter connection.
- Conditionally to the groups of nodes, independence of edges.

Lattice graphes



- Quasi-Homogeneity of degrees.
- May account for a spatially structured network.

Preferential attachment: Barabási-Albert

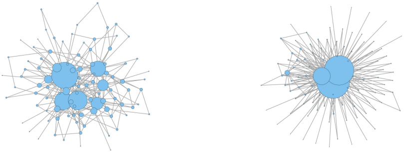


Figure: Preferential attachment networks with attachment power 1 and 3

- A sequentially constructed network.
- An incoming node is linked more likely to the most connected nodes (rich get richer).
- $\mathbb{P}(\cdot \text{ linked to node } k) \propto \text{degree}(k)^{\text{pow}}$.

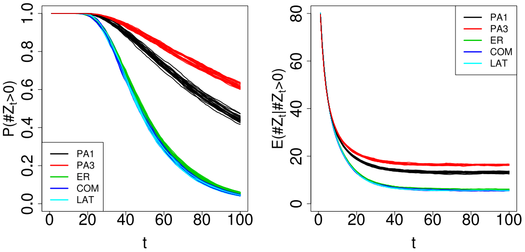
Outline

Sensitivity analysis

$$e,\ c,\ G \rightarrow \boxed{\text{Dynamic Model}} \rightarrow \mathbb{P}(T_0 > 100),\ \mathbb{E}(\#Z_{100})\ ,$$

- main influent parameters are obviously e, c and d the density of G,
- network topology not always important, but can have a key impact for some settings of e, c, d especially when persistence is jeopardized.
- 2 main groups of networks leading to common behaviours
  1. Preferential attachment are more resistant if extinction is probable,
  2. Balanced networks (ER, COM, LAT) have a bigger number of occupancies ($\mathbb{E}(\#Z_{100})$) if extinction is unlikely,
- A network can be better for mean number of occupied patches and worse for the probability of persistence.

An example of the crucial role of the topology in a particular setting



(a) (b)

Figure: (a) Probability of persistence and (b) mean number of occupied patches, in varying t generations (based on 20 replications of the network for a given topology) for $n = 100$, $c = 0.01$, $e = 0.25$ and $d = 30\%$. COM: community network, ER: Erdős-Rényi network, LAT: Lattice network, PA1: preferential attachment network with power 1, PA3: preferential attachment with power 3.

Outline

Scenarios and hypotheses

Networks with density fixed to $p_{50} = 0.21$ and $p_{500} = 0.021$ (constant number of connection per node)

- 1: random seed exchanges among few farmers (ER:50)
- 2: scale-free seed exchanges among few farmers (PA:50)
- 3: community-based seed exchanges among many farmers (COM:500)
- 4: random seed exchanges among many farmers (ER:500)
- 5: scale-free seed exchanges among many farmers (PA:500)

3 levels of event frequency (seed circulation) :

- low frequency $e = 0.1$,
- medium frequency $e = 0.5$,
- high frequency $e = 0.8$.

2 kind of variety :

- popular $c = e$,
- rare $c = e/5$.

Results

Early networks,

	e	$\mathbb{P}(\#Z_{30} > 0)$	$\mathbb{E}(\#Z_{30})$
$e/c = 1$	0.1	$ER = PA = 1$	$ER \sim PA = 44$
	0.5	$ER = PA = 1$	$ER \gtrsim PA = 44$
	0.8	$ER = 0.9 > PA = 0.7$	$ER = 37 > PA = 25$
$e/c = 5$	0.1	$ER = PA = 1$	$PA \gtrsim ER = 25$
	0.5	$PA = 0.8 \gg ER = 0.3$	$PA = 13 \gg ER = 3$
	0.8	$PA = ER = 0$	$PA = ER = 0$

Final networks,

	e	$\mathbb{P}(\#Z_{30} > 0)$	$\mathbb{E}(\#Z_{30})$
$e/c = 1$	0.1	$PA = ER = COM = 1$	$ER \sim COM \gtrsim PA = 425$
	0.5	$PA = ER = COM = 1$	$ER \sim COM \gtrsim PA = 427$
	0.8	$PA \sim ER = COM = 1$	$ER \sim COM = 382 > PA = 314$
$e/c = 5$	0.1	$PA = ER = COM = 1$	$ER \sim COM \sim PA = 249$
	0.5	$ER \sim COM \sim PA = 1$	$PA = 193 \gg ER \gg COM = 40$
	0.8	$PA = 0.5 \gg ER = COM = 0$	$PA = 43 > ER = COM = 0$

Conclusion & Perspectives

Main results:

- Stochastic context with a finite number of patches $\Rightarrow$ finite number of generations studied (chosen accordingly to the application context).
- Most of the times, the role of the topology is not crucial except in cases with high uncertainties.
- Topologies with hubs / central patches are more resistant in case of a likely extinction.
- Community and ER topologies are quite close.

To be continued:

- Refined study on the community topology.
  - different size of communities,
  - different activities,
  - hub in communities.
- Estimation of parameters e, c, G.
- Linking the network with genetic data.

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