Optimization in Energy Markets: the good, the bad and the ugly

Nicolas Gast (Inria)[1]

Journées MAS, Grenoble, 2016

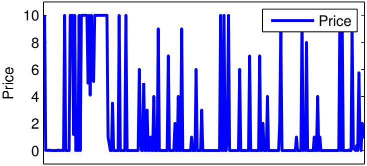
[1] Joint work with Jean-Yves Le Boudec (EPFL), Alexandre Proutiere (KTH) and Dan-Cristian Tomozei (EPFL)

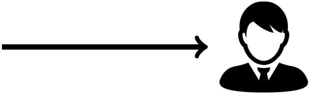
The electrical grid is essentially controlled by markets (for electricity and ancillary services)

- Electricity markets = Day-ahead, real-time (5min)

(Many) people push for more real-time prices (e.g.: at home)

Real-time prices simplifies control





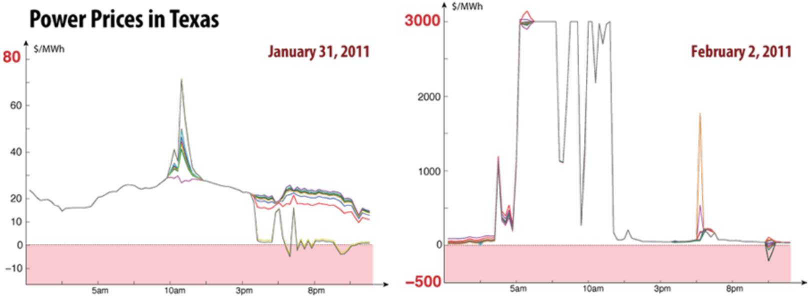
Computes a best response to schedule its appliances (fridges, washing machine, etc.)

Lots of papers

Problem of real-time markets: Is it price manipulation or an *efficient* market?

Source: Meyn 2012.

Prices ($/MWh)



Time of the day Time of the day

Can we develop a mathematical model that captures this behavior?

Question 1. Is there a contradiction between observed prices and "market efficiency"?

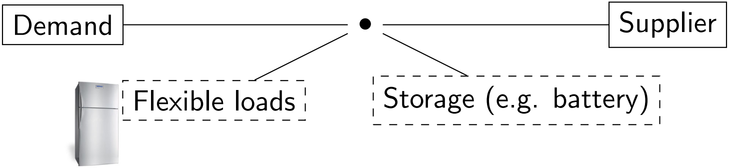
Question 2. Can real-time prices can be used for control?

Outline

1. Market Model
2. Socially optimal allocation and market efficiency
3. Case study: the case of storage
4. Conclusion

Outline

We consider the simplest model that takes the dynamical constraints into account (extension of Wang et al. 2012)



Each player has internal utility/constraints. It exchanges energy.

Examples of internal utility functions and constraints

- Demand:
  - Demand $D(t) = D_f(t) + W(t)$, where W is a Brownian motion.
  - $v \underbrace{\min(D(t), E(t))}_{\text{satisfied demand}} - c^{bo} \underbrace{\max(D(t) - E(t), 0)}_{\text{frustrated demand}}$.
- Supplier: generates $G(t)$ units of energy at time t.
  - Cost of generation: $cG(t)$.
  - Ramping constraints: for all $s > 0$: $s\zeta^- \leq G(t+s) - G(t) \leq s\zeta^+$.
- Storage :
  - No cost for using the storage system
  - Capacity constraint and efficiency η:

    $$0 \leq B_0 + \int_0^t (\eta \mathbf{1}_{u(s)>0} + \mathbf{1}_{u(s)<0}) u(s) ds \leq B_{\max}$$
- Flexible loads: population of thermostatic loads whose consumption can be anticipated/delayed.

We assume perfect competition

Players are selfish and price-takers:

$$\underset{E_i \in \text{internal constraints of } i}{\arg\max} \ \mathbb{E}\left[\int_0^\infty \underbrace{W_i(t)}_{\text{internal utility}} - \underbrace{P(t)E_i(t)}_{\text{bought/sold energy}} dt\right]$$

We assume perfect competition

Players are selfish and price-takers:

$$\arg\max_{E_i \in \text{internal constraints of } i} \boxed{\mathbb{E}} \left[\int_0^\infty \underbrace{W_i(t)}_{\text{internal utility}} - \underbrace{\boxed{P(t)} E_i(t)}_{\text{bought/sold energy}} \, dt \right]$$

Players share a common probabilistic forecast model

Players are price takers:
they cannot influence $P(t)$.

Definition: competitive equilibrium

Each player wants to maximize its expected payoff:

$$\operatorname*{arg\,max}_{E_i \in \text{internal constraints of } i} \mathbb{E}\left[\int_0^\infty \underbrace{W_i(t)}_{\text{internal utility}} - \underbrace{P(t)E_i(t)}_{\text{bought/sold energy}} dt\right]$$

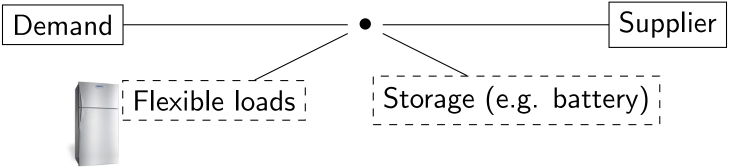
Definition

A competitive equilibrium is a price for which players selfishly agree on what should be bought and sold:

- For any player i, E_i^e is a selfish best response to P:
- For all t: $\displaystyle\sum_{i \in \text{players}} E_i^e(t) = 0.$

Outline

Socially optimal allocation



$$\max_{\substack{E_i \text{ satisfies constraints } i \\ \forall t : \sum_i E_i(t) = 0}} \mathbb{E}\left[\sum_{i \in \text{players}} \int W_i(t) dt\right]$$

The market is efficient (first welfare theorem)

Theorem

For any installed quantity of demand-response or storage, any competitive equilibrium is socially optimal.

If players agree on what should be bought or sold, then it corresponds to a socially optimal allocation.

Proof. The first welfare theorem is a Lagrangian decomposition

For any price process P:

social planner's problem

$$\boxed{\max_{\substack{E_i \text{ satisfies constraints } i\\ \forall t : \sum_i E_i(t) = 0}} \mathbb{E}\left[\sum_{i \in \text{players}} \int W_i(t)dt\right]}$$

selfish response to prices

$$\leq \sum_{i \in \text{players}} \boxed{\max_{E_i \text{ satisfies constraints } i} \mathbb{E}\left[\int (W_i(t) + P(t)E_i(t))dt\right]}$$

If the selfish responses are such that $\sum_i E_i(t) = 0$, the inequality is an equality.

Proof. The first welfare theorem is a Lagrangian decomposition

For any price process P:

social planner's problem

$$\boxed{\max_{\substack{E_i \text{ satisfies constraints } i \\ \forall t : \sum_i E_i(t) = 0}} \mathbb{E}\left[\sum_{i \in \text{players}} \int W_i(t)dt\right]}$$

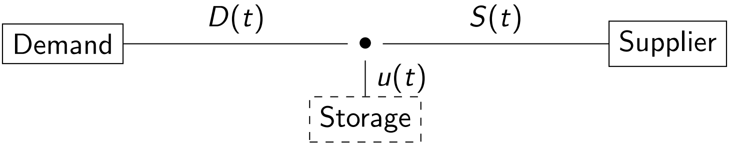
selfish response to prices

$$= \sum_{i \in \text{players}} \boxed{\max_{E_i \text{ satisfies constraints } i} \mathbb{E}\left[\int (W_i(t) + P(t)E_i(t))dt\right]}$$

If the selfish responses are such that $\sum_i E_i(t) = 0$, the inequality is an equality.

Outline

The socially optimal control problem in the case of storage



energy capacity $B_{\max}$,
cycle efficiency η

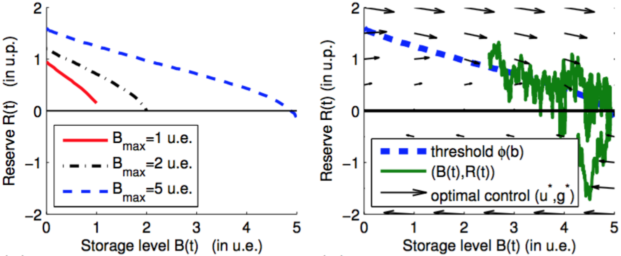
$$\min_{G,u} \mathbb{E}\left[\int ((v + c^{vo}) \underbrace{(D(t) + u(t) - G(t))^{-}}_{\text{frustrated demand}} + cG(t))e^{-\delta t} dt\right]$$

$$\text{such that } \begin{cases} \zeta^{-} s \leq G(t+s) - G(t) \leq s\zeta^{+} \\ B(t) = B(0) + \int_0^t \eta u(t) \mathbf{1}_{u(t)>0} + u(t)\mathbf{1}_{u(t)<0} dt \\ 0 \leq B(t) \leq B_{\max} \end{cases},$$

Structure of the socially optimal control

There exists a decreasing function $\Phi(b)$ such that the optimal control is:

- Increase the generation $G(t)$ if $G(t) - D(t) < \Phi(B(t))$
- Decrease the generation $G(t)$ if $G(t) - D(t) \geq \Phi(B(t))$



(a) Function $b \mapsto \phi(b)$ for various values of the storage energy capacity $B_{\max}$.

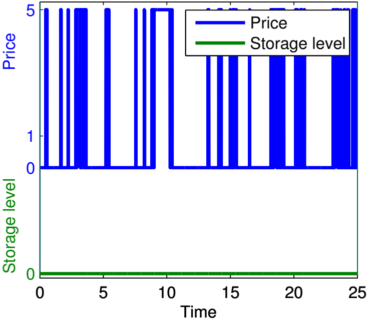
(b) Sample of a trajectory of the optimal reserve and storage processes. $B_{\max} = 5\,\text{u.e.}$

What is the price equilibrium? Is it smooth?

What is the price equilibrium? Is it smooth?

Without storage, the price is never equal to the marginal production cost:

$$P(t) = \begin{cases} 0 & \text{if } G(t) - D(t) > 0 \\ v + c^{bo} & \text{if } G(t) - D(t) < 0 \end{cases}$$

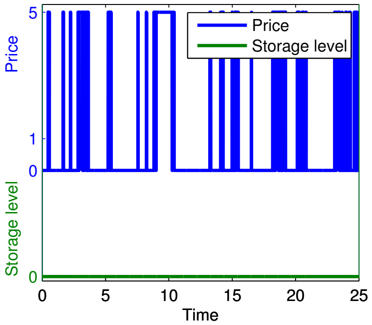


No storage

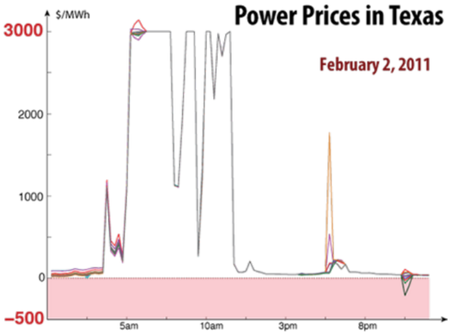
What is the price equilibrium? Is it smooth?

Without storage, the price is never equal to the marginal production cost:

$$P(t) = \begin{cases} 0 & \text{if } G(t) - D(t) > 0 \\ v + c^{bo} & \text{if } G(t) - D(t) < 0 \end{cases}$$



No storage



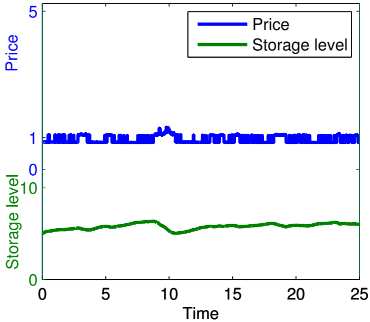
Even with storage, the price is not smooth

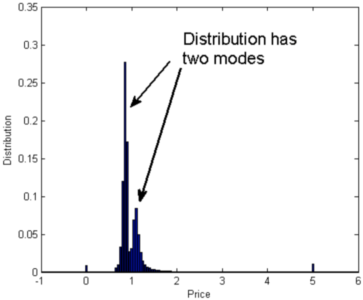
$$P(t) = \begin{cases} 0 & \text{if } G(t) - D(t) > 0 \text{ and } B(t) = B_{\max} \\ \\ \\ v + c^{bo} & \text{if } G(t) - D(t) < 0 \text{ and } B(t) = 0 \end{cases}$$

Even with storage, the price is not smooth

$$P(t) = \begin{cases} 0 & \text{if } G(t) - D(t) > 0 \text{ and } B(t) = B_{\max} \\ \eta \dfrac{\partial}{\partial b} V(G(t) - D(t), B(t)) & \text{if } G(t) - R(t) > 0 \text{ and } B(t) < B_{\max} \\ \dfrac{\partial}{\partial b} V(G(t) - D(t), B(t)) & \text{if } G(t) - D(t) < 0 \text{ and } B(t) > 0 \\ v + c^{bo} & \text{if } G(t) - D(t) < 0 \text{ and } B(t) = 0 \end{cases}$$

where $V(s-d, b)$ is the value function.

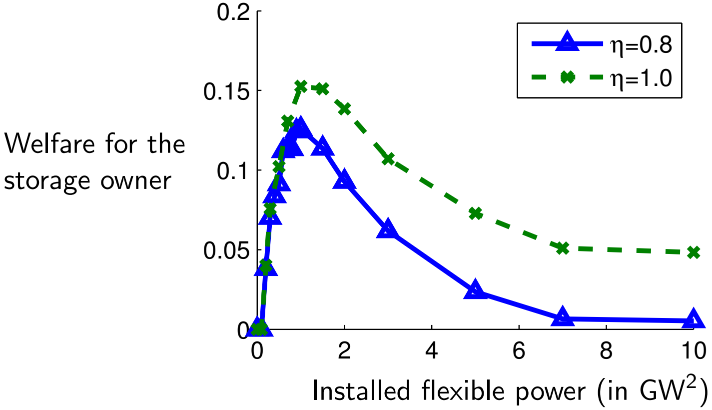




Two modes in $\sqrt{\eta}$ and $1/\sqrt{\eta}$

The invisible hand of the market may not be optimal

With a fixed storage capacity, a competitive equilibrium leads to an optimal use of the resources. Yet, there is incentive to install less storage than the social optimal



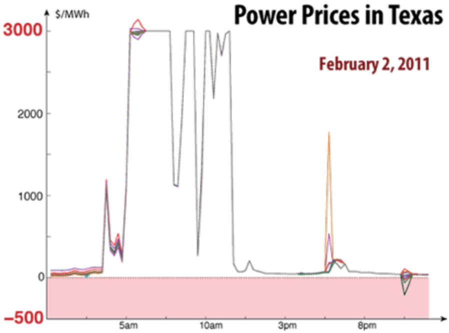
[2] The forecast errors correspond to a total wind capacity of 26GW.

Outline

The good Observed prices are not incompatible with the model of an efficient market.

The bad Prices are highly volatile

The ugly The market structure is not good for investment



Do you want this at home?