Absence of percolation in a family of germes-grains model

Simon Le Stum,
Joint work with D.Coupier and D.Dereudre

Journées MAS, Grenoble, august 29-th 2016

1 Introductive model and definitions

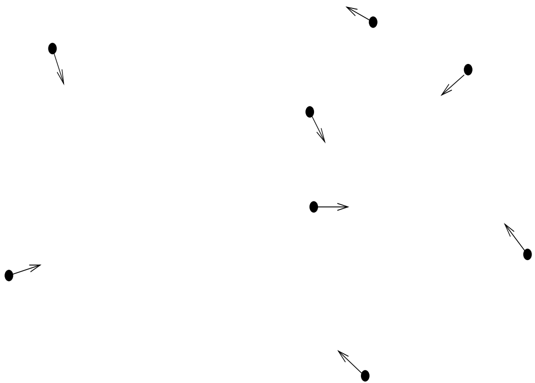
Marked configuration

$\mathbb{R}^d$ the euclidian space.
$\mathbb{M} \subset \mathbb{R}$ compact, called space of mark, and $\mathcal{Q}$ a probability law on $\mathbb{M}$.

Definition (Space of marked configuration)

Let $\varphi \subset \mathbb{R}^d \times \mathbb{M}$ and $\Pi : \mathbb{R}^d \times \mathbb{M} \longrightarrow \mathbb{R}^d$ the projection.
We say that φ is a marked configuration if $\Pi(\varphi)$ is locally finite in $\mathbb{R}^d$ and the restriction of Π to φ is injective. We note by $\mathcal{C}^{\mathbb{M}}$ the set of marked configuration.

Example 1



The Model

Definition (graph function)

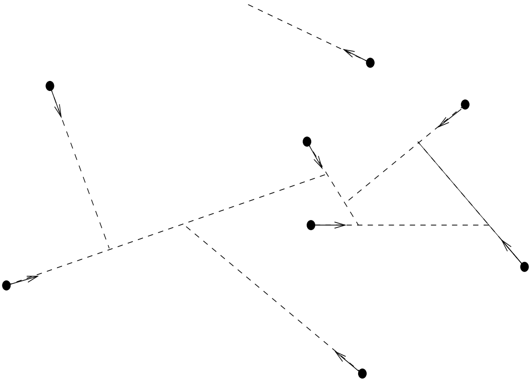
Let $\mathcal{C}' \subset \mathcal{C}^{\mathbb{M}}$, we call **graph function** or **building graph function**, each function :

$$h : \mathcal{C}' \times (\mathbb{R}^d \times \mathbb{M}) \longrightarrow \mathbb{R}^d \times \mathbb{M} \text{ such that :}$$

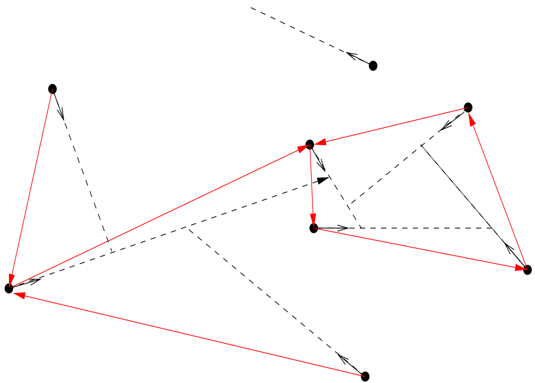
(i) $\forall \varphi \in \mathcal{C}', \forall x \in \varphi \,;\, h(\varphi, x) \in \varphi$.
(ii) $\forall \varphi \in \mathcal{C}', \; \forall \overrightarrow{v} \in \mathbb{R}^d$, we have $\varphi + \overrightarrow{v} \in \mathcal{C}'$ and $h(\varphi + \overrightarrow{v}, x + \overrightarrow{v}) = h(\varphi, x) + \overrightarrow{v}$.

A given couple $(\mathcal{C}', h)$ is called **random graph model** if the realisations of a stationary and independently marked Poisson point process live almost surely in $\mathcal{C}'$.

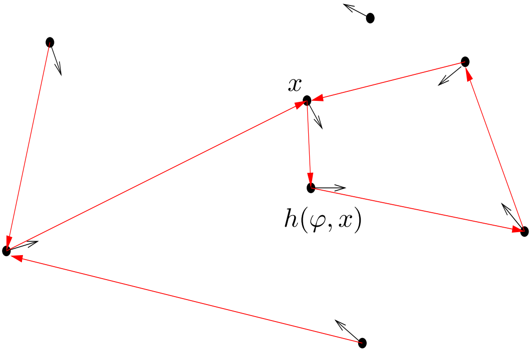
Dynamic on the example 1



Graph function on the example 1



Graph function on the example 1



Cluster

Definition

Let $\varphi \in \mathcal{C}'$ and $x \in \varphi$, we define the forward of x in φ :

$$For(x,\varphi) := \{x,\ h(\varphi,x),\ h(\varphi,h(\varphi,x))...\}.$$

We also define the backward of x in φ :

$$Back(x,\varphi) = \{y \in \varphi :\ x \in For(y,\varphi)\}.$$

To finish, we introduce $C(x,\varphi) = For(x,\varphi) \cup Back(x,\varphi)$.

2 Assumptions and theorem

Cycle Assumption

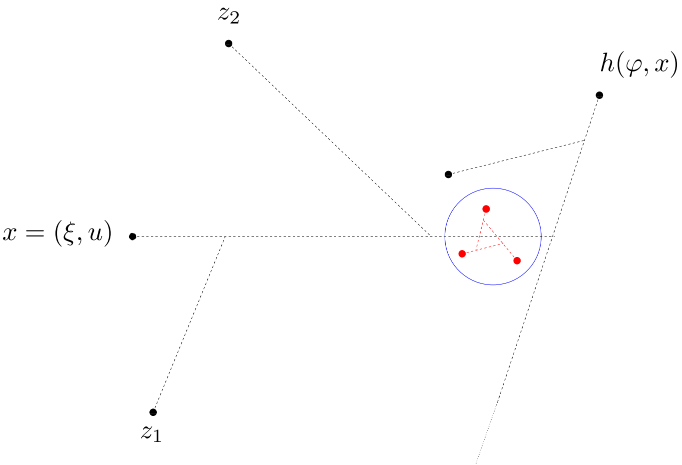
Definition

Let $\varphi \in \mathcal{C}'$ and $k \in \mathbb{N}$, we say that φ is a **k-cycling configuration** if :
$\forall x \in \varphi,\ \exists A_x \subset (\mathbb{R}^d \times \mathbb{M})^k$ an open ball such that :
$\forall (x_1, ..., x_k) \in A_x$, then :
(i) $For(x, \varphi \cup \{x_1, ..., x_k\}) = \{x, x_1, ...x_k\}$.
(ii) $\#Back(x, \varphi \cup \{x_1, ..., x_k\}) \geq \#Back(x, \varphi)$

Cycle assumption There exists $k \in \mathbb{N} \setminus \{0\}$ such that :

$$\mathbb{P}[\mathbb{X}\ is\ a\ k - cycling\ configuration] = 1$$

Cycle assumption for the line segment model



Shield assumption

Shield Assumption

There exists $\alpha \in \mathbb{N} \setminus \{0\}$ and $(E_m)_{m\geq 1}$ a sequence of events, such that :

(i) $\forall m \geq 1$, E_m is $B(0, \alpha m)$ measurable.

(ii) $\mathbb{P}[E_m] \underset{m \to +\infty}{\longrightarrow} 1$.

(iii)$\forall V \subset \mathbb{Z}^d$ such that $\mathbb{Z}^d \setminus V$ contains at least two connected components A_1 and A_2 such that :

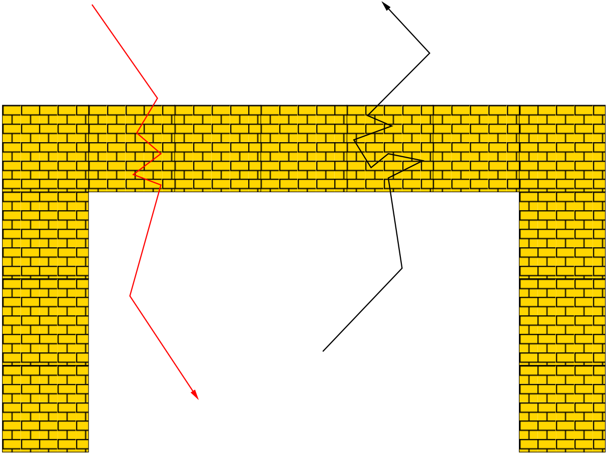
$\forall i \in \{1,2\},\ \mathcal{A}_i := (A_i \oplus [\frac{-1}{2}, \frac{1}{2}]^d) \setminus (V \oplus [-\alpha, \alpha]^d) \neq \emptyset$.

then, for m sufficiently large :

$\forall \varphi, \varphi' \in \mathcal{C}'$ such that : $\forall z \in V,\ \varphi - mz \in E_m$ (we say that z is a m**-shield vertex** for φ), then :

$$\forall x \in \varphi_{m\mathcal{A}_1},\ h(\varphi, x) = h(\varphi_{m\mathcal{A}_2^c} \cup \varphi'_{m\mathcal{A}_2}, x).$$
$$\forall x \in \varphi_{m\mathcal{A}_2},\ h(\varphi, x) = h(\varphi_{m\mathcal{A}_1^c} \cup \varphi'_{m\mathcal{A}_1}, x).$$

Shield assumption for the line segment model



Theorem

Let $(\mathcal{C}', h)$ a random graph model relatively to a given Poisson point process $\mathbb{X}$.

Theorem

Let $(\mathcal{C}', h, \mathbb{X})$ a random graph model satisfying the two assumptions, then :

$$\mathbb{P}[\forall x \in \mathbb{X}, \#C(x, \mathbb{X}) < \infty] = 1$$

3 Main steps of the proof

No percolation backward

In all the rest of the presentation, $(\mathcal{C}', h, \mathbb{X})$ is a fixed random graph model satisfying our two assumptions.
Using the mass transport principle, we have :

Lemma

If we suppose that $\mathbb{P}[\forall x \in \mathbb{X},\ \#For(x, \mathbb{X}) < \infty] = 1$, *then*

$$\mathbb{P}[\forall x \in \mathbb{X},\ \#Back(x, \mathbb{X}) < \infty] = 1$$

Now, we have to proof that $\mathbb{P}[\forall x \in \mathbb{X},\ \#For(x, \mathbb{X}) < \infty] = 1$.

Looping point

Definition (Looping point)

Let $0 < r < R < \infty$, $K \in \mathbb{N} \setminus \{0\}$, $\varphi \in \mathcal{C}'$ and $x \in \varphi$. We say that x is a **Looping point** of φ if :
(i) $\#\varphi_{B(x,R)} \leq K$.
(ii) $For(x, \varphi)$ loop inside of the ball $B(x, r)$.

Using the mass transport principle, we have :

Lemma

For all choices of parameters (r, R, K) in the Looping point definition, we have :

$$\mathbb{E}[\#Back(\Theta, \mathbb{X}_\Theta)\mathbb{1}_{\{\Theta \textit{ is a Looping point of } \mathbb{X}_\Theta\}}] < \infty$$

Θ is a typical point of $\mathbb{R}^d \times \mathbb{M}$ and we use the notation $\mathbb{X}_\Theta$ for $\mathbb{X} \cup \Theta$.

A-Looping point

Definition

Let $0 < r < R$, $K \in \mathbb{N} \setminus \{0\}$, $A \subset (B(0,r) \times \mathbb{M})^k$ an open ball, $\varphi \in \mathcal{C}'$ and $x \in \varphi$. We say that x is a **A-Looping point** of φ if :
(i) $\#\varphi_{B(x,R)} \leq K$.
(ii) $\forall (x_1, ..., x_k) \in A_x$, then :

$$For(x, \varphi \cup \{x_1, ..., x_k\}) \text{ is cycling in } B(x,r).$$
$$\#Back(x, \varphi \cup \{x_1, ..., x_k\}) \geq \#Back(x, \varphi)$$

Where the subset A_x is obtained using a translation operator on the subset A.

Connection between Looping point and A-Looping point

Lemma

If there exist parameters (r, R, K, A) *such that,*

$$\mathbb{E}[\#Back(\Theta, \mathbb{X}_\Theta \mathbb{1}_{\{\Theta \text{ is } (r,R,K,A)-\textbf{looping} \text{ for } \mathbb{X}_\Theta\}}] = \infty.$$

Then,

$$\mathbb{E}[\#Back(\Theta, \mathbb{X}_\Theta)\mathbb{1}_{\{\Theta \text{ is } (r,R,K+k)-\textbf{looping} \text{ for } \mathbb{X}_\Theta\}}] = \infty$$

Proof conclusion

Proposition

*Let $(\mathcal{C}', h)$ a random graph model satisfying k-**cycle assumption** and **shield assumption**. If we suppose that :*

$$\mathbb{P}[\{\#For(\Theta, \mathbb{X}_\Theta) = \infty\}] > 0$$

then, there exists parameters (r, R, K, A) such that :

$$\mathbb{E}[\#Back(\Theta, \mathbb{X}_\Theta)\mathbb{1}_{\{\Theta \text{ is a } (r,R,K,A)-\text{looping point of } \mathbb{X}_\Theta\}}] = \infty$$

Thank you for your attention